Physics IV
ISI B.Math
Midterm Exam : February 27,2012

Total Marks: 60
Time : 3 hours
Answer all questions

1. **Marks : 4 + 5 + 6 = 15**
Two trains A and B, each have proper length L and move in the same direction. A's speed with respect to the ground is $\frac{4c}{5}$, and B's speed is $\frac{3c}{5}$. A starts behind B. C is an observer standing on the ground.
(a) How long, as viewed by C, does it take for A to overtake B ?

(b) How long, as viewed by A does it take for A to overtake B? How long does it take for A to overtake B as viewed by B?

(c) Let the event E_1 be " the front of A passing the back of B ". Event E_2 : " The back of A passing the front of B". Person D, walks at constant speed from the back of B to the front of B, such that he coincides with both events E_1 and E_2. How long does the "overtaking" process take, as viewed by D?

2. **Marks: 2 + 2 + 2 + 2 = 8**
State whether the following are true or false with a one or two line justification :

(i) Two clocks at the extreme ends of a train are synchronized with respect to the train frame. You are standing on the ground and the train moves past you. To you, the clock at the front of the train will show a later time compared to the clock at the back.

(ii) An observer in Delhi reports a bomb explosion at 12 pm and another observer reports a bomb explosion in Kolkata at 2pm. It is not possible to find an inertial reference frame where the two explosions occur at the same time.

(iii) A man sits in a train holding a 1m rod in his hand such that the rod is perpendicular to the floor of the train. The train moving with a speed $\frac{\sqrt{3}}{2}c$. To a man standing on the platform, the rod will appear to be 0.5 m long.

(iv) The total energy of an isolated electron at rest is zero

3. **Marks : 2 + 3 + 3 + 3 + 3 = 14**
Draw the ct and x axis of the spacetime coordinates of the observer $\mathcal{O}$. Then draw
(a) the world line of $\mathcal{O}$'s clock at $x = 1m$.

(b)ct' and x' axes of an observer $\mathcal{O}'$ who moves with a velocity $v = 0.5c$ in a positive x direction relative to $\mathcal{O}$ and whose origin ($x' = ct' = 0$) coincides with that of $\mathcal{O}$.

(c) the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = - 1 m^2

(d)the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = 0.

(e)the locus of events, all of which occur simultaneously at $ct = 2m$ according to $\mathcal{O}$.

4. **Marks 6 + 4 + 5 = 15:**
(a) Show that the the sum of two future-pointing null four-vectors is either timelike or null, and is again future-pointing. Under what conditions is the sum null?

(b) Every four-vector orthogonal to a timelike vector is spacelike. (Two four vectors A^μ and B^μ are orthogonal if $A^\mu B_\mu = B^\mu A_\mu = 0$ $\mu = 0, 1, 2, 3$)

(c) If u^μ is the four-velocity of a particle and $a^\mu = \frac{du^\mu}{d\tau}$ is the four-acceleration (where τ is the proper time), show that $u^\mu a_\mu = 0$.

5. **Marks: 8**

A photon collides with a stationary electron. If the photon scatters at an angle θ ,use the conservation of 4-momentum to show that the resulting wavelength λ' is given in terms of the original wavelength λ of the photon, by

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where m is the rest mass of the electron. Note: The energy of a photon is $E = h\nu = \frac{hc}{\lambda}$. [Hint: Use the fact that $P_\mu P^\mu = m_0^2 c^2$ for a particle of rest mass m_0 and 4-momentum P^μ]